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EQUIVALENCE OF THE IMPEDANCE METHOD AND THE METHOD OF AMPLITUDE-FREQUENCY CHARACTERISTICS FOR INVESTIGATIONS OF VIBRATIONS IN HYDRAULICALLY POWERED SUPPORTS

N. A. Dokukova[a] **and P. N. Konon**[b]

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Two approaches to investigation of hydroelastic shock absorbers ensuring an efficient decrease in vibrations and noise in mobile machines have been presented. It has been shown that the parameters obtained by the method of amplitude-frequency characteristics more accurately correspond to the parameters of the physical model than the data computed with the impedance method.

Hydroelastic technologies of damping of vibrations and noise, embodied in a new generation of shock absorbers — hydraulically powered supports — have found application at the current stage of development of mobile equipment. Such elements have been analyzed theoretically in the works of Russian and foreign authors [1–3]. The investigation procedure is based on the application of the methods of chain dynamic systems or total mechanical resistance, i.e., impedance. These methods enable one to analyze motions and forces acting on each part of the physical system, including the system consisting of "black boxes" [4], which leads to a simplified mathematical model of the problem since the forces and motions at one or two points of the system are determined without analyzing the entire mechanical system completely. The simplicity of the analysis of the physical model involves the neglect of certain regularities exerting a substantial influence on the dynamics of the object under study, i.e., the hydraulically powered support. Another method of investigation of dynamic systems that takes the most complete account of these circumstances is the method of amplitude-frequency characteristics [5]. It is based on the mathematical formalization of all the possible motions of the object and the hydraulically powered support and on solution of the system of linear or linearized equations by the operator method. This enables one to analyze the mechanical system with the use of transfer functions and frequency characteristics and to obtain the solutions sought from the Riemann–Mellin formulas.

We demonstrate the application of the above methods using the dynamic model of vibroinsulation of an automobile by hydraulically powered supports. This model is represented in the form of a two-mass mechanical system with a hydraulically powered support possessing a dry c_1 and viscous b friction (Fig. 1). The wheel of the automobile is replaced by the mass m_2; its tire has an elasticity c_2. The hydraulic shock absorber is intended to damp vibrations of mass m_1 that are produced by the harmonic force $F_1(t) = f_1 \cos \omega t$ and to insulate against vibrations transferred by the base of a roadway covering $F_2(t) = f_2 \cos \nu t$.

The impedance Z of a moving point and the mobility of mechanical systems $M = 1/Z$ with lumped parameters without account for the masses m_1 and m_2 and the external action of the road are expressed by the formulas [4]

$$Z = \frac{b - i\left[\frac{b^2\omega}{c_2} + \frac{c_1}{\omega}\left(\frac{c_1 + c_2}{c_2}\right)\right]}{\left(\frac{c_1 + c_2}{c_2}\right)^2 + \left(\frac{b\omega}{c_2}\right)^2}, \tag{1}$$

[a]Institute of Mechanics and of the Reliability of Machines, National Academy of Sciences of Belarus, Minsk, Belarus; [b]Belarusian State University, Minsk, Belarus. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 76, No. 6, pp. 174–176, November–December, 2003. Original article submitted April 16, 2003.

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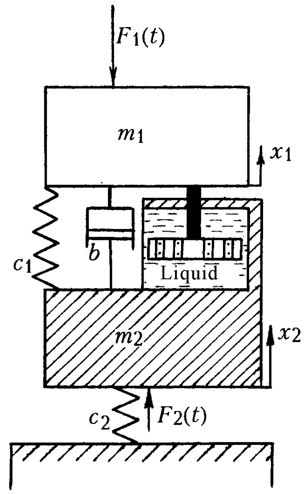

Fig. 1. Dynamic scheme of the automobile suspension with a hydraulically powered support.

$$M = \frac{b + i\left[\frac{b^2\omega}{c_2} + \frac{c_1}{\omega}\left(\frac{c_1 + c_2}{c_2}\right)\right]}{\left(\frac{c_1}{\omega}\right)^2 + b^2} . \tag{2}$$

With allowance for the above factors and methods of calculation of the impedances of parallel and series connection of the elements the dependence of the total mechanical resistance becomes much more complex ($k = m_1/m_2$):

$$Z(i\omega) = -i\,\frac{((b^2\omega^2 + c_1^2)[m_1\omega^2 - (k+1)c_2] + (c_1 + ib\omega)\,m_1c_2\omega^2)\,m_1c_2\omega}{b^2\omega^2[m_1\omega^2 - (k+1)c_2]^2 + [c_1c_2(k+1) - m_1\omega^2(c_1 + c_2)]^2} . \tag{3}$$

To implement the method of amplitude-frequency characteristics we write a complete system of the equations of motion of the mechanical system whose diagram is presented in Fig. 1:

$$\ddot{x}_1 = 2n_1(\dot{x}_2 - \dot{x}_1) + \omega_1^2(x_2 - x_1) + f_1\cos\omega t\,,\quad \ddot{x}_2 = 2n_2(\dot{x}_1 - \dot{x}_2) + \omega_3^2(x_1 - x_2) - \omega_2^2 x_2 + f_2\cos\nu t\,. \tag{4}$$

The sign of f_1 is taken into account in the numerical calculations.

The system of linear differential equations (4) with initial conditions $\dot{x}_1(0) = 0$, $\dot{x}_2(0) = 0$, $x_1(0) = 0$, and $x_2(0) = 0$, as a result of the application of the integral Laplace transformation, is reduced to the system of linear algebraic equations from which we find the sought inverse transforms of the complex amplitudes $A_1 = X_1(p)$ and $A_2 = X_2(p)$:

$$X_1(p) = \frac{\tilde{f}_1(p^2 + k(2n_1p + \omega_1^2) + \omega_2^2) + \tilde{f}_2(2n_1p + \omega_1^2)}{p^4 + p^2((1+k)(\omega_1^2 + 2n_1p) + \omega_2^2 + 2n_1p) + \omega_1^2\omega_2^2}\,,$$

$$X_2(p) = \frac{k\tilde{f}_1(2n_1p + \omega_1^2) + \tilde{f}_2(p^2 + 2n_1p + \omega_1^2)}{p^4 + p^2((1+k)(\omega_1^2 + 2n_1p) + \omega_2^2 + 2n_1p) + \omega_1^2\omega_2^2}\,, \tag{5}$$

here $\tilde{f}_1$ and $\tilde{f}_2$ are the Laplace transforms of the functions $f_1\cos\omega t$ and $f_2\cos\nu t$.

The moduli of the complex quantities $A_1 = X_1(i\omega)$ and $A_2 = X_2(i\omega)$ in harmonic excitation by the force $f_1\cos\omega t$ are the amplitudes of movement of the masses m_1 and m_2 or the amplitude-frequency characteristics. The dynamic transfer functions have the form $W_1(i\omega) = X_1(i\omega)/\tilde{f}_1$ and $W_2(i\omega) = X_2(i\omega)/\tilde{f}_1$. Their inverse quatities

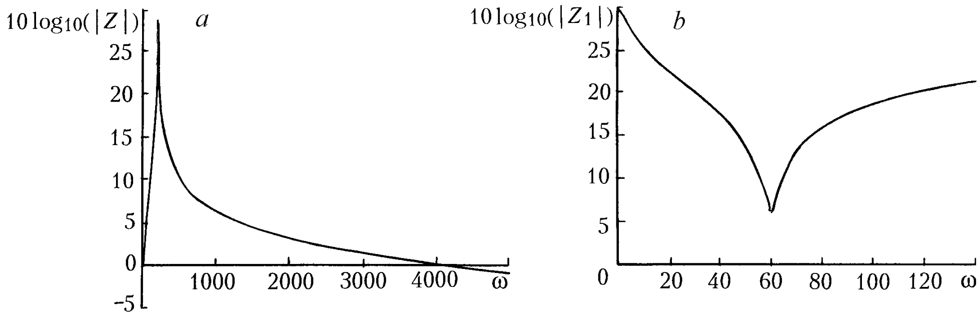

Fig. 2. Logarithmic dependence of the impedance 10 $\log_{10}(|Z|)$ (a) and 10 $\log_{10}(|Z_1|)$ (b) on the frequency ω for the following parameters of the mechanical system: $m_1 = 1$, $c_1 = 100$, $c_2 = 3960$, $k = 10$, and $b = 900$.

$$D_1(i\omega) = \tilde{f}_1 / X_1(i\omega)\,, \quad D_2(i\omega) = \tilde{f}_1 / X_2(i\omega) \tag{6}$$

correspond to the parameter of dynamic resistance in impedance theory. Replacing the movements in the denominators of these quantities by the velocities, we obtain the characteristics of the impedances:

$$Z_1(i\omega) = -\frac{i\tilde{f}_1}{\omega X_1(i\omega)}\,, \quad Z_2(i\omega) = -\frac{i\tilde{f}_1}{\omega X_2(i\omega)}\,. \tag{7}$$

For $\tilde{f}_2 = 0$ the impedance $Z_1(i\omega)$ will be as follows:

$$Z_1(i\omega) = i\,[\omega(m_1\omega + ikb) - k(c_1 + c_2)] \times$$

$$\times \frac{(m_1\omega^2(k+1) - kc_2)(m_1\omega^2 - (k+1)c_1 - ib\omega) - k^2 c_1 c_2 - km_1\omega^2(m_1\omega^2 - (k+1)c_1)}{\omega(b^2\omega^2 + (c_1 + c_2)^2)}\,. \tag{8}$$

The impedances computed with two different approaches to an analysis of one mechanical system from formulas (3) and (8) do not coincide in the complex region of the parameter ω in absolute value (Fig. 2). The procedure of amplitude-frequency characteristics follows from the complete analysis of the dynamics of the considered mechanical system written on the basis of the well-known Lagrange equations of the second kind, the D'Alembert principle, and the second Newton law. Therefore, the question arises as to whether it is expedient to use the impedance method for investigations and calculations of elastic hydraulic shock absorbers. The replacement of the five-element scheme (see Fig. 1) by the equivalent two-element scheme according to formula (3) does not lead to exact results. Dependences (3) and (8) become equal only in the case of the limiting transitions in them; in the first case, one and the same parameter m_1 tends to infinity, which physically means a rigid base ($m_2 = m_1/k$), while in the second case m_1 tends to zero.

The amplitude-frequency characteristics for the same coefficients are presented in Fig. 3. Passing to the limits in formula (3) when $m_1 \to \infty$ and in (7) and (8) when $m_1 \to 0$, we obtain

$$Z = \lim_{m_1 \to \infty} Z(i\omega)\,, \quad Z_1 = \lim_{m_1 \to 0} Z_1(i\omega)\,, \quad Z_2 = \lim_{m_1 \to 0} Z_2(i\omega) = -i\,\frac{c_2}{\omega}\,; \tag{9}$$

$$\lim_{m_1 \to \infty} Z(i\omega) = \lim_{m_1 \to 0} Z_1(i\omega) = \frac{b - i\left(b^2\dfrac{\omega}{c_2} + \dfrac{c_1}{\omega}\left(1 + \dfrac{c_1}{c_2}\right)\right)}{\left(1 + \dfrac{c_1}{c_2}\right)^2 + b^2\dfrac{\omega^2}{c_2^2}}\,. \tag{10}$$

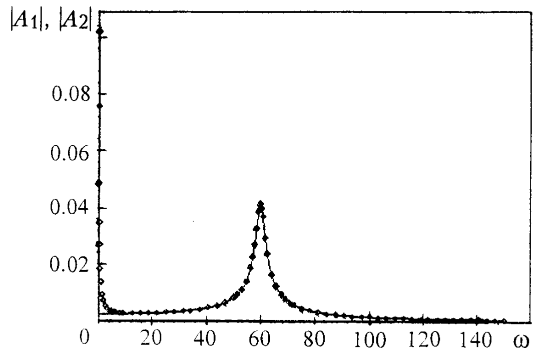

Fig. 3. Amplitudes $|A_1|$ (points) and $|A_2|$ (solid curve) vs. frequency ω ($\tilde{f}_1 = 10$, $\tilde{f}_2 = 1$).

Thus, the two procedures become identical only in the case of simplification of the scheme to three elements: they are confirmed by the reference data in [4] and coincide with formula (1).

Based on the analysis made, we should note that the method of total mechanical resistance is rather simple to apply to multielement mechanical structures; it enables one to replace complex structures by an equivalent scheme with two basic components, the ohmic resistance and the reactance, and is convenient for the purpose of preliminary investigations. It is more expedient to use the method of amplitude-frequency characteristics to obtain reliable results on the vibration loading of hydraulically powered supports.

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NOTATION

c_1 and b, coefficients of dry and viscous friction; m_1, reduced part of the automobile mass; m_2, mass of the automobile wheel, its tire has a coefficient of elasticity c_2; ω, frequency of vibrations of the external force F_1; ν, frequency of vibrations transferred by the base of the roadway covering, it is expressed by the force load F_2; $2n_1 = b/m_1$; $2n_2 = b/m_2 = 2kn_1$; $\omega_1^2 = c_1/m_1$; $\omega_3^2 = c_1/m_2 = k\omega_1^2$; $\omega_2^2 = c_2/m_2$; $k = m_1/m_2$; n_i, sec^{-1}; ω_i, rad·sec^{-1}; F_i, N; c_i, kg·sec^{-2}; b, kg·sec^{-1}; m_i, kg; $|A_i|$, m; $|Z_i|$, kg·sec^{-1}; k, dimensionless parameter; $i = 1, 2$. Subscripts: 1 corresponds to the mass m_1; 2 corresponds to the mass m_2.

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